HANDS ON!

A magazine for mathematics
and science educators

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FALL 2020

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Computational Thinking
and Executive Function

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Letter from the President

It seems like a lifetime ago that the Spring 2020 *Hands On!* came out—we were at the beginning of the COVID19 pandemic and had yet to grasp how learning would be impacted. At TERC, our researchers may not be physically in classrooms or attending in person conferences, but they continue to promote equal access to learning opportunities for all and are growing their work in alternate ways. We are excited to share it with you.

Jodi Asbell-Clarke's INFACT project (*Including Neurodiversity in Foundational and Applied Computational Thinking*) developed out of their research observing how students build CT skills from their use of video games, and educators' reflections on how different types of learners engage with CT. *Computational Thinking and Executive Function: Where Neurodiversity Shines* demonstrates the overlap between neurodiversity and technology-related problem solving, two "hot areas" in education that may have more in common than first meets the eye. Read how INFACT is building tools that prepare students for a computational world and also support executive function so each learners' unique strengths can shine.

Introducing the Signing Bioscience Dictionary (SBD) by Judy Vesel, et al., addresses the need for a four-year interpreter training program to increase access to science content for deaf and hard of hearing undergraduate students. Their research supports the benefits of student interpreters learning science vocabulary and their need for more support to become skilled in using that vocabulary to interpret science content. As a result, more students who are deaf or hard of hearing may pursue STEM fields and ultimately choose a STEM career or a career that involves STEM.

Mathematical argumentation happens when students engage, as a classroom community of learners, in deciding together what is mathematically true or false. In *Mathematical Argumentation, Open-ended Conjecturing, and Equity*, Jennifer Knudsen shows how she and her team have supported a practice that fosters creativity, inclusiveness, and equity, developing professional development that aims to ease teachers and students into the argumentation process.

During these challenging times, I am especially proud to work with these amazing colleagues and very appreciative of your readership.

Laurie

Laurie Brennan, *President*

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Cover photo by Sam Felder

Computational Thinking

and Executive Function

WHERE NEURODIVERSITY SHINES

By Jodi Asbell-Clarke

Educators understand more and more these days that each student's brain works a little bit differently. Every learner has unique cognitive strengths (or assets) and some weaknesses (or deficits). Parents know that each child learns and plays differently too. Some children express themselves readily through art or music, some are fascinated by the natural world outdoors, while others are delighted by an entire afternoon with a difficult jigsaw puzzle.

As schools serve increasingly diverse student populations, the need for educators to differentiate learning activities to meet the needs of their students is growing tremendously (Immordino-Yang & Darling-Hammond, 2018). Adapting a lesson to engage all students—including those with learning issues related to neurology (e.g., ADHD, autism, or dyslexia)—and to keep them persistent and productive in their tasks is not easy. It requires considering the cognitive assets and deficits of each child to leverage learners' strengths to support them while they power through tougher assignments. Educators need support to deliver classroom approaches that are inclusive and draw on the unique strengths of neurodiverse learners (Tomlinson, C. A., & Strickland, 2005). In particular, technology such as video games may play a key role in supporting learners with diverse needs (Goodwin, 2008; Parsons, Leonard, & Mitchell, 2006).

Neurodiverse learners' tendency toward systematic behavior and compulsion for detail, labeled in school as a "learning disability" related to cognitive inflexibility, can be seen as exactly the skillset needed to thrive in a computational world (Abraham, Windmann, Siefen, Daum, & Güntürkün, 2006; Dawson et al., 2007; Schmidt & Beck, 2016; White & Shah, 2011). Many IT companies, such as Microsoft, have specific hiring programs for neurodiverse people, because the companies understand the unique capabilities these employees bring to the table for tasks such as quality assurance and debugging software. Divergent thinking and impulsive reactions that might be seen as disruptive to classrooms could be just what a design team needs to break through a rut in problem-solving.

What is Computational Thinking? Hands On! Spring 2020. https://blog.terc.edu/what-is-computational-thinking

This overlap between neurodiversity and technology-related problem solving has led our team to study the intersection between Computational Thinking (CT) and Executive Function (EF). These are two "hot areas" in education and may have more in common than first meets the eye. Our current project INFACT (Including Neurodiversity in Foundational and Applied Computational Thinking) developed out of our research observing how students build CT skills from their use of video games, and educators' reflections on how different types of learners engage with CT. We are now building tools that prepare students for a computational world and also support executive function, so each learners' unique strengths can shine.

Computational Thinking

Computational Thinking has been discussed in education since the mid 1990s and is now being adopted in many state standards (CSTA, 2017; Shute, Chen & Asbell-Clarke, 2017; Wing, 2006). There are numerous programs aiming to teach CT, from pre-school through adult classes. CT can be thought of as the set of practices used when humans solve problems similarly to how computers solve problems. It involves devising and classifying problems that could have similar solutions, then building sets of instructions (algorithms) for solving groups or classes of problems, rather than solving each new problem from scratch. CT practices include:

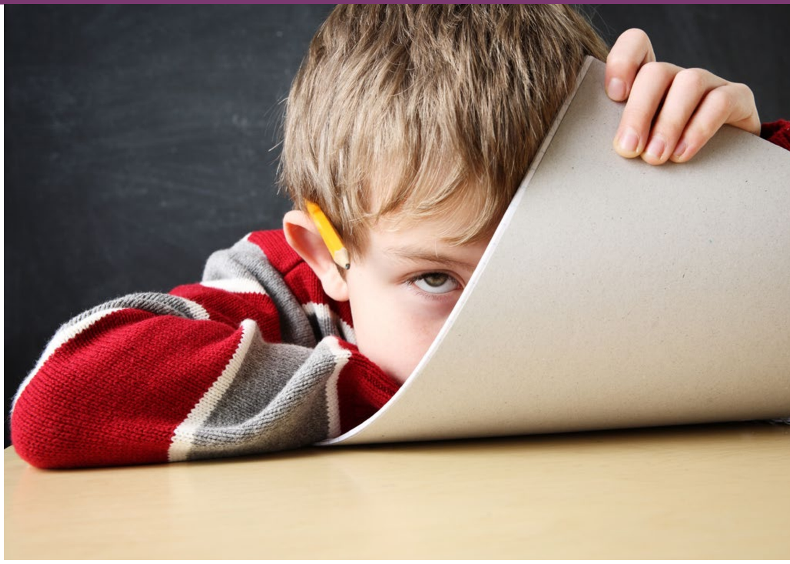
- **Problem Decomposition:** breaking up a complex problem into smaller, more manageable problems;
- **Pattern Recognition:** seeing patterns among problems that may have similar types of solutions;
- **Abstraction:** generalizing problems into groups by removing the specific information and finding the core design of each problem; and
- **Algorithmic Thinking:** thinking of problem-solutions as a set of general instructions that can be re-used in different settings.

While a natural application of CT is coding (computer programming), there are many learning activities and uses for CT without a computer. When considering CT as a mode of problem-solving, one can see many applications of CT even in daily life. For example, writing a recipe or designing an instruction manual for a piece of equipment is sometimes described as a CT activity. Recipes and manuals could be seen as algorithms—sets of instructions to be implemented by another user.

We would argue, however, that a handwritten recipe card from your grandparent with instructions for their famous sweet and sour chicken (for instance) is not an algorithm, because it does not demonstrate the concept of abstraction that is core to CT. Abstraction is about generalizing instructions (here, a recipe) to provide the basic structure that a user can apply to a variety of contexts. An abstracted recipe (or algorithm) could describe how a chef makes a sweet and sour sauce. In this case, we see the structure:

- one third something savory
- one third something tangy
- one third something sweet

This general pattern is an algorithm that is re-usable with different ingredients. In one case the chef may use soy, lemon, and honey; and in another case they may use herbs, vinegar, and sugar. But even for folks who are not aspiring chefs or computer programmers, CT may be a useful way to think about how our brains work.



Executive Function

Executive Function (EF) is rapidly being recognized as a key area of focus for education for all learners, not just those in special education (Immordino-Yang & Darling-Hammond, 2018; Meltzer, 2018). A neurological description of EF usually includes:

- **Working Memory:** how we store information in the short term as we are solving a problem;
- **Cognitive Flexibility:** how well we can express and modify our thinking when provided new information; and
- **Inhibitory Control:** how well we can squelch tendencies to do things we shouldn't do, and focus on the things we should do.

Psychologists and educators consider the social and emotional aspects of executive function including emotional regulation, motivation, and metacognitive processes like planning a task, organizing steps and information, and monitoring progress. An educational perspective of EF refers to how these processes play out in the classroom with regard to students' ability to:

- retain information while reading a passage or solving a word problem;
- express their thinking and refine their ideas with experience;
- focus and navigate their way through a task; and
- manage frustration and regulate emotions.

CT and Executive Function

Over the past several years, our team has been studying how learners in grades 3-8 build CT practices through games such as TERC's popular logic puzzle game, *Zoombinis* (available at zoombinis.com). We also partnered with a Massachusetts school district in a Research-Practice Partnership to infuse CT into their classroom curriculum for grades 3-8. Throughout our research, we found teachers observing that some learners who struggled in other subjects became more engaged and more productive when doing CT activities—sometimes even becoming leaders in their class.

The struggles of many learners in school boil down to issues with EF. The practices of CT —breaking down problems into smaller pieces and finding patterns in problems so that they can generalize solutions— are also practices that support EF. They help learners to focus and navigate their way through tasks and to refine their ideas with experience.

We also found that special education teachers were excited by teaching CT, because, as one put it, "These are the problem-solving skills I always try to teach our kids, and now I have words for it. And I have a way to embed it right in the curriculum." Teachers with English Language learners noted the same thing: CT helped them support their learners by making learning explicit. Teachers saw this type of success spill over into other areas by the building of student confidence and social capital as well as academic skills. These observations led us to study the intersection between EF and CT.

Examples of How to Support EF in CT

CT can be seen as useful strategies for solving problems of all kinds, particularly when encountering similar problems or tasks over and over again. Calling out and emphasizing CT practices may help support EF in other areas.

For a simple illustration, let's think about solving a jigsaw puzzle. Most people who have done a few puzzles establish a set of routines or repeatable procedures—algorithms—when they sit down to do a new puzzle. Steps for building, using, and modifying those algorithms relate to CT practices.

Decomposition

A 1,000-piece jigsaw puzzle may seem like a daunting task at first, but when you break it up into sub-tasks it becomes more manageable. There are many ways puzzle solvers decompose the problem, such as working on the edge first before tackling the interior, or choosing one region of the puzzle to work on at a time.

Pattern Recognition

Many people sort pieces by color, while others look at the shapes of pieces and the number of "innies" and "outies". These patterns help the problem become more manageable and provide information that makes the puzzle solution more apparent.

Abstraction

Puzzle solvers may begin to generalize about types of pieces, for example by collecting all those that have "outies" side by side before finding pieces that exactly fit together. In this systematic method, the solver doesn't have to try each piece every time; they only have to start by finding pieces that fit the general category of "outie".

Algorithmic Thinking

An expert jigsaw puzzler might always put the edge together first, then group by color, then sort those piles by shape before they assemble. Their problem-solution can be thought of as a set of general instructions that can be re-used in different settings and that helps them develop fluency in puzzle solving.

These same practices can be thought of as ways to support EF in problem solving. Table 1 shows the relationship between CT practices, jigsaw puzzle-solving activities, and EF.

Table 1

CT Practice	Jigsaw Puzzle Activities	EF Support
Problem Decomposition	Breaking down puzzle into regions or types of pieces	Smaller problems are easier on the working memory.
Pattern Recognition	Grouping pieces by shape or color	Seeing patterns can help with cognitive flexibility by relating one context to another.
Abstraction	Searching for general types of pieces to fit into places	Generalizing solutions can simplify problems which can help with retaining information, and focused navigation through a task.
Algorithmic Thinking	Developing re-usable routines to solve the puzzle	Developing an algorithm helps with explicit thinking and task navigation.

Designing Supports for Neurodiverse Learners

INFACT is using these ideas to design, implement, and study a comprehensive and inclusive CT program to support teachers and students for grades 3-8. It focuses on the cognitive assets of neurodiverse learners and builds in supports for learners with a wide variety of differences in attention, metacognition, and self-regulation. The materials engage learners' EF within CT activities to help make learners' thinking visible and their problem-solving productive. Learn more at https://www.terc.edu/projects/infact/

For example, we are building a flashlight tool that highlights relevant information that a learner might not be attending to, so that they can focus on the salient information in an activity. We are designing graphical organizers that help learners keep track of necessary information, removing the load on their working memory, and helping them organize the information in ways that make meaning. We are also providing an expression tool that helps learners make their implicit thinking visible, so they can see exactly what they've done in one task and re-use similar strategies for future problem solving.

Currently we are designing these supports within digital learning activities, such as games like *Zoombinis*, so that we will be able to use data mining algorithms to make the supports adaptive. We are building models to detect when students are getting overly frustrated or bored and where in the activities they are no longer productive. When students persist unproductively, it is called wheel-spinning and can lead to disengagement. By detecting in real-time the "trigger" points just before wheel-spinning starts, we are planning to intervene with a "just-in-time" support—like suggesting a strategy they've used previously, highlighting useful information they might be missing, or suggesting they take a break and come back after re-energizing. Finding ways to react to each learner's levels of engagement and potential wheel-spinning through automated data mining detectors will allow us to support individual learners' unique needs.

CT activities offer unique opportunities to support EF, and in turn by supporting EF we strive to improve learners' CT. This symbiosis of these two areas may show us a way to help create a world where learners with many different cognitive assets and challenges will thrive. And where our society will benefit from the creativity and intelligence that all learners have to offer.

Acknowledgements

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Author

Jodi's academic background includes an MA in Math, an MSc in Astrophysics, and a PhD in Education. Early in her career, Jodi dreamed of being an astronaut and went to Houston where she was an onboard software verification analyst for IBM during the first 25 missions of the space shuttle. Later Jodi taught Physics and Astrophysics to some of the brightest students in the country at the laboratory school at University of Illinois. In 1993, she came to TERC where she led several science education projects involving curriculum development, professional development of teachers, and educational research. In 2009, she co-founded EdGE with her colleagues to study how game-based learning can transform science education.

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Introducing the Signing Bioscience Dictionary (SBD)

BY JUDY VESEL, M. DIANE CLARK, & TARA ROBILLARD

Why Is the SBD Needed?

Students who are deaf or hard of hearing are not necessarily considered “print disabled.” However, difficulties in acquisition of English literacy skills often arise due to early language delay. This results in considerable literacy limitations that lead to the majority of deaf students leaving high school with a reading level at the fifth grade or below. In fact, the English vocabulary of the average 15-year-old deaf child is about the size of that of the average 9-year-old hearing child and will not improve significantly (Qi & Mitchell, 2012).

Since most students who are deaf or hard of hearing and who use sign for communication are in classrooms that do not use sign language, interpreters are a frequent accommodation. Given this situation, the interpreter's skills need to be taken into consideration. Most Interpreter Training Programs (ITP) focus on fluency in American Sign Language. However, most students enrolled in ITP programs have little or no science background and are not required to take general education courses in science as part of the curriculum (Graham et al., 2012). Those interpreters who are available may fail to make the language "visible" or comprehensible for learners and may rely heavily on fingerspelling and word-for-word transliteration, thereby rendering science course content minimally accessible (Seal, Wynn, & MacDonald, 2002).

This lack of formal and standardized training results in those students who require interpreting services in science classes being confronted by a scarcity of qualified and trained interpreters. Specifically, students in science courses frequently receive content translation from interpreters who are unfamiliar with concepts or do not have a command of the necessary specialized vocabulary needed for accurate interpretations (Gormally, 2017). When asked, interpreters often mention the need for opportunities for engaging in training related to providing services in the science disciplines. Many of them state that familiarity with and understanding of discipline-specific vocabulary usually happens by default—on the job (Grooms, 2015).

Complicating this state of affairs is the historic lack of a common sign-language lexicon for scientific terms. In recent years, several online databases that include ASL signs for technical scientific terms and concepts have become available. Among these are TERC's Signing Math & Science Dictionaries. Although these were developed for grades K-12, we knew that interpreters were using the dictionaries

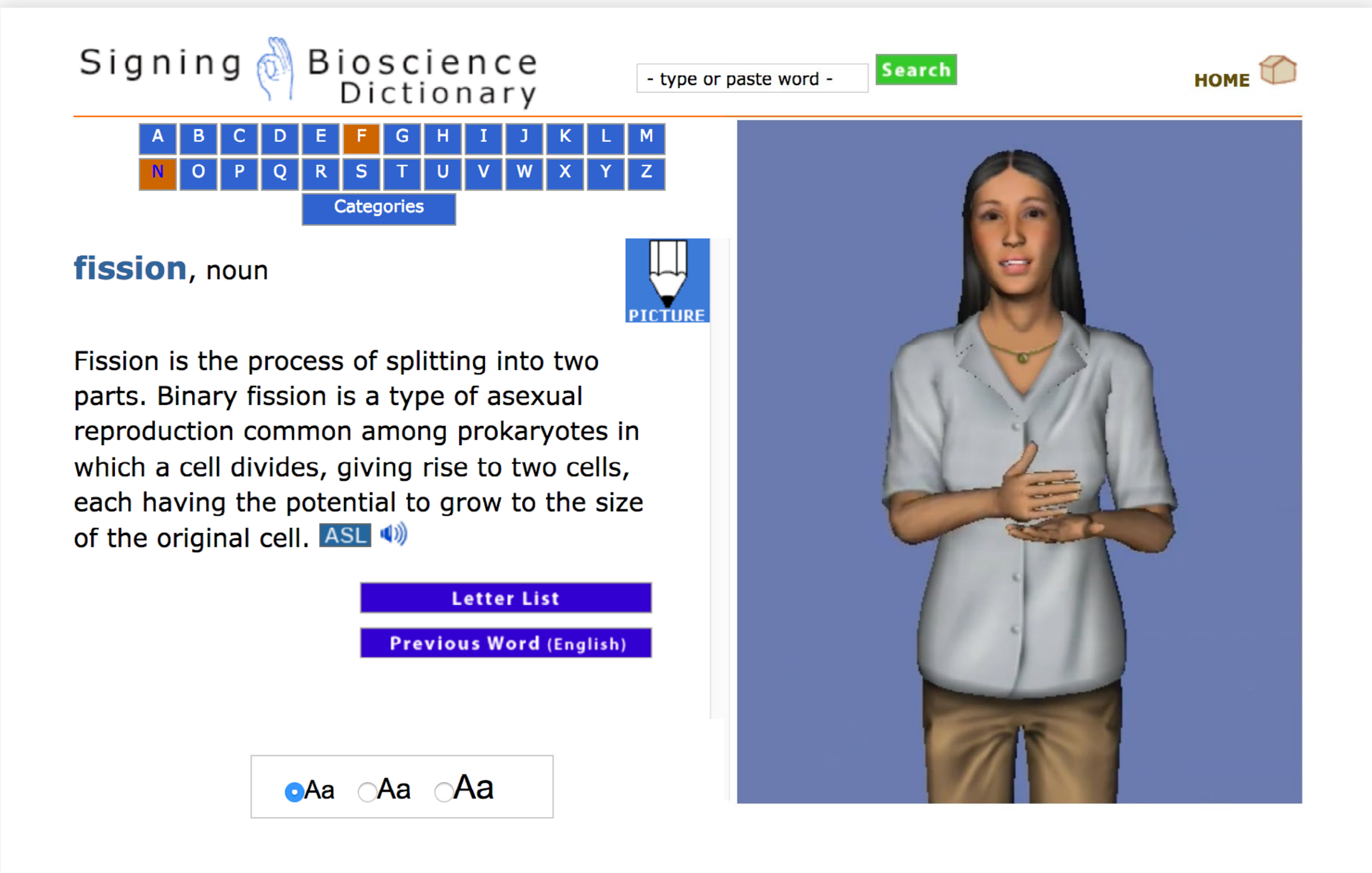

Figure 1. An Example of a Dictionary Page

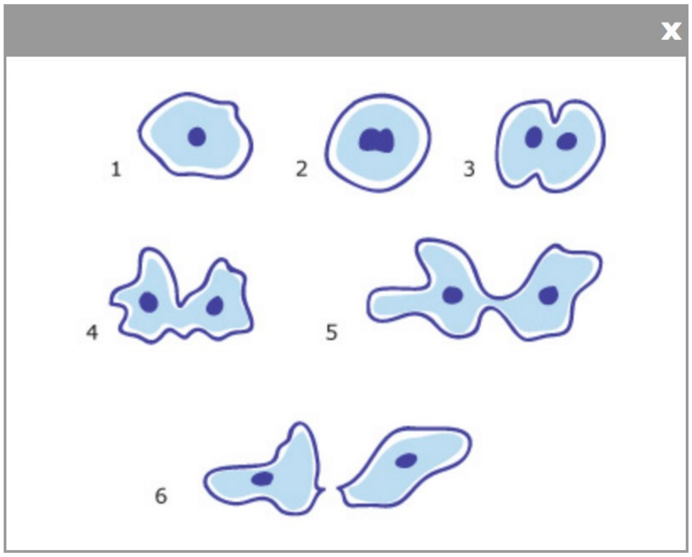

Figure 2. An Example of an Illustration Page

to help them interpret undergraduate science material (Vesel & Robillard, 2014). This realization pointed to the need for a signing dictionary that specifically focused on terms used in undergraduate science courses. Working in partnership, TERC and Lamar University, with funding from NSF, developed such a dictionary in the form of a prototype Signing Bioscience Dictionary (SBD). We then conducted a first-of-its kind study to examine the use and effectiveness of the SBD in helping Lamar ITP students develop a robust technical vocabulary and interpret science content fluently and accurately.

What Terms and Features Have Been Incorporated into the SBD?

Development of the SBD involved the Lamar team in reviewing the glossary entries in Campbell Biology, 8th Edition to identify an initial set of terms. This text was selected because it is used at the university for their undergraduate biology courses. The review resulted in a list of terms, organized by text chapter, that were submitted to the TERC team, who then identified terms from the Lamar list that also appear in the signing dictionaries. The partners then identified additional terms that were not in the Lamar list but that are necessary for fully understanding the meaning of a signing dictionary term. The result was a final list of 1,580 terms.

The Lamar team then used the Campbell Biology chapter headings to create content categories for the terms in the final list. Review of the additional terms drawn from the signing dictionaries, with respect to their fit with a category, resulted in a final set of 12 categories. TERC adapted the existing interface for the signing dictionaries to create an SBD that is compatible with a wide variety of devices, platforms, and Web browsers.

As is the case with the signing dictionaries, the SBD is "universally designed" according to the Universal Design for Learning (UDL) framework (Rose & Meyer, 2006). This type of design means that the interface incorporates interactive features to offer users multiple means of representing information, multiple means for expression of knowledge, and multiple means of engagement in learning. They can access each definition page by typing into a search box or selecting from alphabetical or category lists. They can select information represented as static images, text, human-voice narration and/or signing; they can increase or decrease size of the text; they can view a range of avatar characters. Figures 1 and 2 are examples of pages from the SBD.

What Does Our Research Show?

To evaluate use and effectiveness of the SBD, 28 interpreting students enrolled in Lamar's ASL program used the SBD with each of three units: Reproduction; Heredity & Genetics; Ecology & Ecosystems. Prior to using the SBD, students were introduced to the unit of study. They then completed a Matching Vocabulary and Definition Pre-test, a Signing Pre-test, and a Pre-interpreting sample. Each instrument focused on terms for the unit that are found in the SBD. Students were then shown how to use the interactive SBD features. They used the SBD over the course of several weeks to practice signs, and while studying the meaning of terms on their own and while researchers observed them. At the end of the unit, they completed a Participant Survey that sought information about their perceptions regarding SBD use. They also completed a Matching Vocabulary and Definition Post-test, a Signing Post-test, and an Interpreting sample—all of which were the same as the pre-unit versions. This procedure was followed for each of the three units of study.

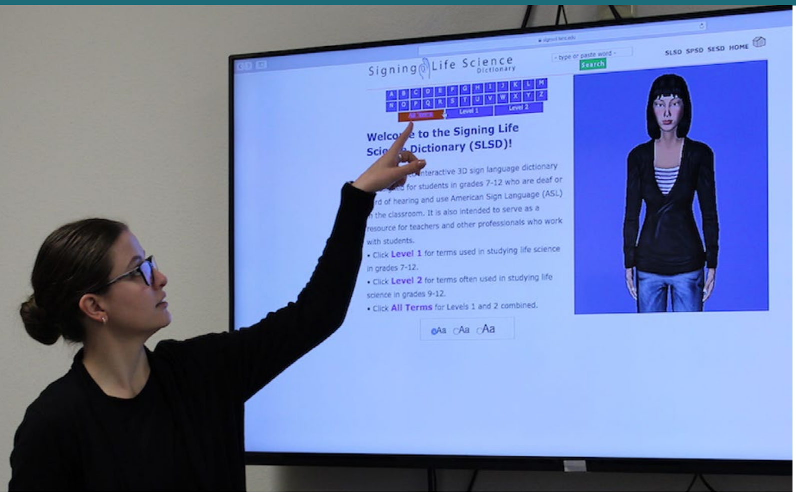

Figure 3. Introducing the SBD

Results from the survey and differences between pre- and post-use scores supplied answers to the following four primary research questions:

1. How do Lamar undergraduate interpreting students use the SBD to learn life science terms?
2. How effective is the SBD in increasing Lamar undergraduate interpreting students' knowledge of the vocabulary and their ability to sign life science terms?
3. How effective is the SBD in increasing Lamar undergraduate interpreting students' capacity to accurately and clearly interpret content typically taught in undergraduate biology courses?
4. What additions and/or changes would make the SBD more effective?

We learned from observing interpreting students and from their responses to survey questions that most students found the SBD helpful and easy to use, had fun using it, and would use it again. They used the interactive features to look up terms in ASL and English, see words signed, view illustrations, learn new signs, and learn more about life science. All participants found that using the dictionary made learning science terms and definitions easier and helped them learn on their own. They were generally satisfied with the information that was available for each term, with the accuracy of the signs, with their ability to understand the avatar, and with the avatar's facial expressions. Some preferred a human signer to an avatar or thought the avatar was difficult to understand. Additionally, some found the avatar's signing choppy. Some felt the contrast between the avatar's clothing and skin color to be insufficient and that this interfered with seeing the signing. These results will be used to make changes to the SBD to render it more effective.

We also learned that SBD use increased students' knowledge of the life science content and related vocabulary presented in the definitions, and that they were able to learn this vocabulary rather quickly. However, much of this vocabulary includes fingerspelling, and it became clear that many of the students were not skilled in using morphology to "chunk" items to make them easier to sign. In addition, they were unable to fingerspell terms and then set up an expansion using depiction constructions or classifiers, which are ASL structures that establish a pronoun-like referent, to convey the information in a fluent manner. Depiction constructions are representations of nonliteral behaviors in a visual-spatial format. Their use allows abstract ideas to become visual.

Figures 4 & 5. Practicing Signing of SBD Terms

Given the complexity of life science content, interpreting students need to master these skills in order to be effective and fluent interpreters for deaf students.

Current research (NSF Award #2019843) is examining involvement of ITP students in developing these ASL skills. For example, they can be explicitly taught to chunk fingerspelling into morphemes to increase ease of comprehension through modelling and training, thereby enabling them to sign more like native signers when they fingerspell. In addition, native signers can develop examples of expansions using depiction constructions so that interpreting students gain deeper comprehension by connecting the fingerspelling of a word to its expansion. For example, when the functions of the body are being described, depiction constructions act as a surrogate of a human body and provide a step-by-step process that is easy to follow. By viewing native signers using these constructions, and then being trained in how to use constructions in interpreting situations, interpreters should be able to more effectively communicate life science lecture and classroom content. However, further research is needed to verify this.

In conclusion, this study found that student interpreters benefit from learning science vocabulary but need more support to become skilled in using that vocabulary to interpret science content. It is critical to address this need, as interpreters often accept a job that involves science interpreting, do it once, and then decline subsequent jobs because they find they do not have the necessary skills to interpret science well.* Addressing this need during a four-year interpreter training program will likely increase access to science content for deaf and hard of hearing undergraduate students. As a result, more students who are deaf or hard of hearing may pursue STEM fields and ultimately choose a STEM career or a career that involves STEM.

Where Are Downloads Available?

For more information about our research and to download the SBD, visit our project website at https://signsci.terc.edu/video/SBD_index.html

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Credits

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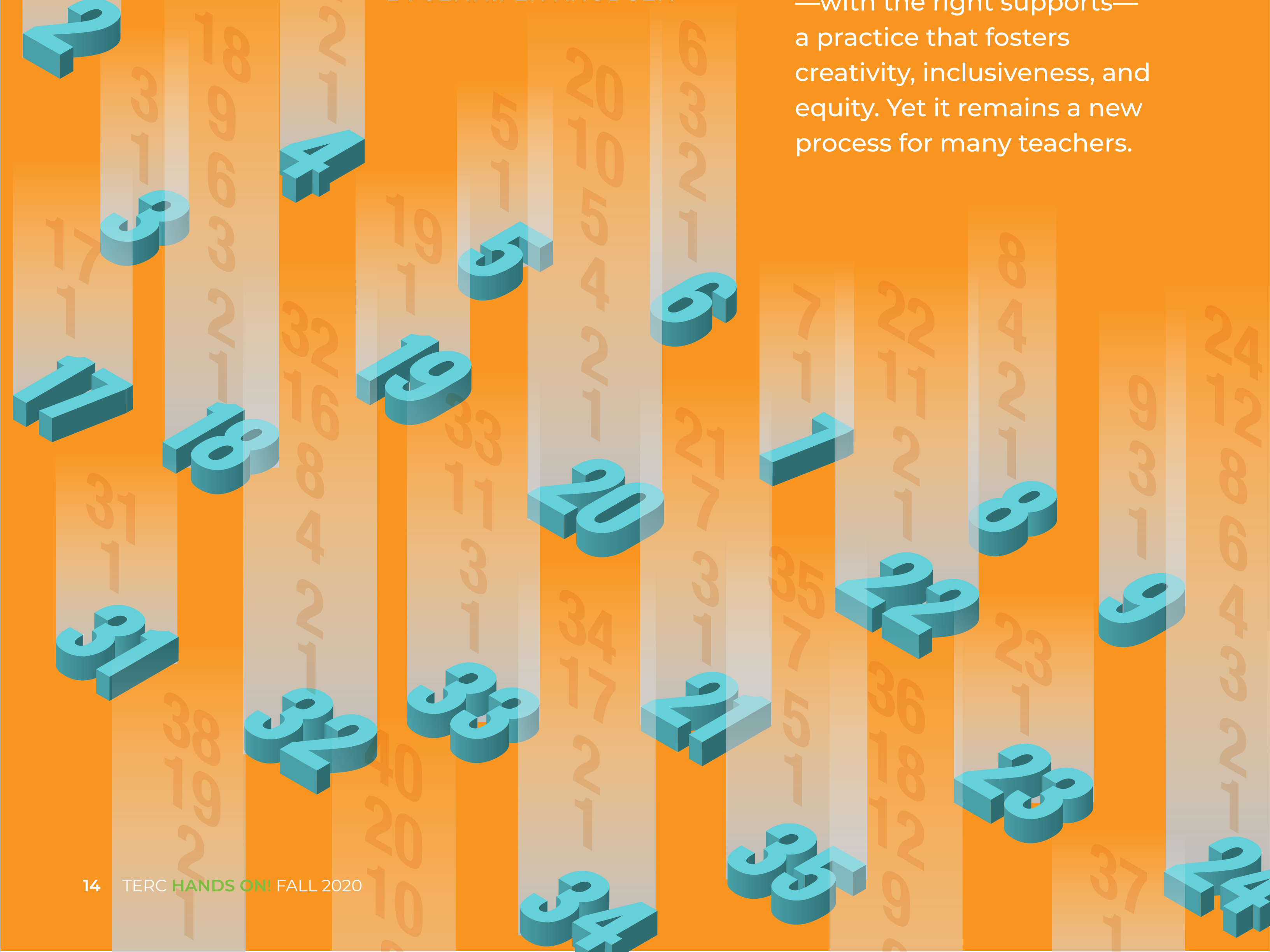
* from S. Barnett, personal communication, February 6, 2020

Mathematical Argumentation, Open-ended Conjecturing, and Equity

BY JENNIFER KNUDSEN

Mathematical argumentation happens when students engage, as a classroom community of learners, in deciding together what is mathematically true or false. Argumentation has been integrated into many state standards over recent decades, and it can be —with the right supports— a practice that fosters creativity, inclusiveness, and equity. Yet it remains a new process for many teachers.



Our project team has developed professional development (PD) that aims to ease teachers and students into the argumentation process. In this article, we explain our model for middle-school argumentation, our PD model, and how they work to support equitable teaching practices.

We define the four parts of argumentation as a vehicle for dividing classroom discourse into clearly defined phases. In our book *Mathematical Argumentation in Middle School-The What, Why, and How* (Knudsen, et al., 2017) we describe each phase:

- **Generating cases:** creating something to argue about
- **Conjecturing:** making bold claims
- **Justifying:** building a chain of reasoning
- **Concluding:** getting closure on truth or falsity

In practice, argumentation gets messier than that. For example, in a live classroom you might get rapid cycles of conjecturing and justifying, interspersed with new cases being generated. However, we've have found that these four phases are distinct enough to simplify the process for teachers and students new to argumentation.

Open-ended Conjecturing

Trying out conjecturing is a great way to get a feel for argumentation. In a recent webinar, as part of the National Council for Mathematics Teachers' 100 Days of Professional Learning, our team engaged about 400 teachers in open-ended conjecturing—the process of a group making multiple, novel (to them) conjectures about a mathematical situation. We presented them with a table of numbers from 1 to 40 with a complete list of the factors of a number below it, and the square numbers highlighted in orange (see Table 1). The teachers were asked to make their best guesses about what might be true about all the whole numbers, up to and beyond 40. The numbers highlighted in orange offered starting points for successful conjecturing.

Below are three conjectures offered by the teachers, which though distinct, turn out to be related.

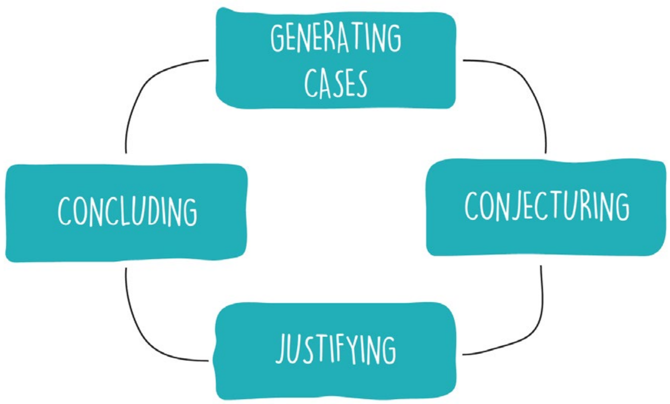
- The square factor is the median of the factors listed.
- Squares have an odd number of factors.
- Non-squares have even numbers of factors.

How are these conjectures related? Take the first one: if you remember or look up the definition of median, you'll quickly deduce that when the median of a data set is actually one of data points in that set, then the data set must contain an odd number of data points (otherwise the median is the average of two points). Here the data set is the list of factors. So, if the "square factor" of a number (the whole-number square root of that number) is on the list of factors, then that square number must have an odd number of factors. For example, 9 has the factors 1, 3, 9. Three is the middle number—the median. Three is the square root of 9. So, 9 is a square number and it

Table 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	3	2	5	2	7	2	3	2	11	2	13	2	3	2	17	2	10	2
			4		3		4	9	5		3		7	5	4		3	19	4
					6		8		10		4		14	15	8		6		5
											6				16		9		10
											12						18		20

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	2	23	2	5	2	3	2	29	2	31	2	3	2	5	2	37	2	3	2
7	11		3	25	13	9	4		3		4	11	17	7	3		19	13	4
21	22		4		26	27	7		5		8	33	34	35	4		38	39	5
			6				14		6		16				6				8
			8				28		10		32				9				10
			12						15						12				20
			24						30						18				40
															36				



Four part argumentation model

has an odd number of factors. Of course, we can simply make this list of three factors for the number 9, to establish that 9, a perfect square number, has an odd number of factors. But in argumentation, we are searching for what *always* must be true—in this case, for any square number.

These three conjectures were made in the context of open-ended conjecturing, based on a mathematical situation or task that affords multiple conjectures. In practice, we recommend sticking to the conjecturing phase until quite a number of conjectures have been made, with a focus on what *always might* be true, and only then moving into justifying. Yet, in this particular example, the lines between conjecturing and justifying are blurred, because one conjecture can be used to justify another—that's how it can work out in the classroom, too.

There are many other conjectures that can be made based on this table about square numbers, prime numbers, and even twin primes (with only one number between them). Try it yourself and have fun!

Of course, open-ended conjecturing is not just for teachers, but something we strongly advocate for students. The table of factors above comes from our book mentioned earlier (Corwin 2017) and is aimed at 6^{th} graders. However, more commonly students are given single conjectures to justify: for example, "The product of two fractions less than one is smaller than either factor." That's a fine conjecture to explore. Opportunities for students to make their own conjectures, beyond a simple "agree" or "disagree", are found less frequently, yet the open-ended process can yield rich results. For example, the table of factors has been successfully used online with 5th graders who were just beginning to understand the concept of factors.

Conjecturing for Equity

Why is open-ended conjecturing, and argumentation overall, considered to be supportive of classroom-level equity? One of the equitable teaching practices advocated by Aguirre, Mayfield-Ingram, and Martin (2013) is "going deeper with the mathematics." Open-ended conjecturing is precisely how mathematics teachers and students can do this. A table of whole numbers can yield multiple, quite sophisticated conjectures—there were over 200 made at our presentation. The task offers multiple entry points: students can learn what a factor is from engaging in the task, considering the factors as a data set, or using algebraic symbols to express their conjectures (all entry points used by teachers in our presentation). These multiple entry points also help ensure equity, as students from different backgrounds, with different approaches and levels of entry-knowledge, can participate fully and creatively in the activity.

Scholars also promote creativity as a way to make mathematics more inclusive for students who are Black, Indigenous, or people of color (BIPOC) (Gutierrez, 2018). As well, helping students to speak up (in this case, to give voice to their own conjectures) supports students' mathematical agency, which is key to classroom equity (Cobb & Hodge, 2007). Our PD provides strategies for teachers to support students to use their voices, improvise creatively, and go deeper into mathematics.

Games and Improvisations for Teaching Moves and Norms

Engaging students in all four parts of argumentation— from a set of cases, to conjectures about them, to justifications of the conjectures, to stating general conclusions—can be challenging. It requires a specialized set of teaching moves and norms. That's what our PD is designed for: to help teachers establish norms and develop teaching moves that support students' participation in argumentation.

Our current PD model includes a series of successive "approximations of practice" (Grossman, et al, 2009): engaging in mathematical argumentation as learners; choosing teaching moves to support argumentation among students; playing "teaching games" to try out and practice new moves; and engaging in "visualization planning." This sequence enables teachers to slowly acquire expertise in new practices.

Two activities unique to our PD model are teaching games and visualization planning.

Teaching games (Knudsen & Shechtman, 2016) are akin to activities that improvisational theater performers use to learn their craft. We provide well-specified rules to bound teaching moves that are improvisationally deployed. A teacher and their peers play out part of an argumentation lesson, with a facilitator observing. The game is then discussed to determine the usefulness and challenges of the particular moves that were in play.

Open- and Closed-Ended Questions

In this game, teachers are given a conjecture to justify.

- In the first round they are allowed to ask only open-ended questions, such as "Why do you think so?"
- In the second round, they may ask only closed-ended questions, such as "Which numbers on the list are square numbers?"

After both rounds, teachers compare the advantages and limitations of each type of question. In live conjecturing in the classroom, we suggest teachers use both types, not just one. This game helps them to know the potential purposes of each.

Visualization planning is a process that pairs a teacher and another teacher, the teacher's coach, or a PD leader. They choose a part of an argumentation lesson on which to focus. The teacher sits back and imagines out loud how that part of the lesson might go, anticipating questions they might ask and students' likely responses. Their partner asks probing questions, helping the teacher envision different routes the lesson might take. The partner also records the imagining in a lesson-planning form that the teacher can take away and elaborate on.

Our PD makes unique use of "improv" games as metaphors for social and socio-mathematical norms—the norms that set a productive atmosphere for argumentation and that students must share in order to work collaboratively as a community of mathematicians on an argument (Nussbaum, 2008). Improv games provide a way to internalize norms through experiential learning, in a way more likely to "stick" than simply by placing them on a poster on the wall. To demonstrate, let's look at a game called Gift Giving.

Gift Giving*

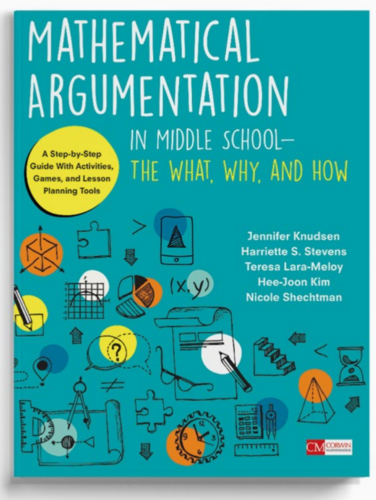
In this game, players work in pairs.

- The first player reaches into an imaginary closet, pulls out an imaginary wrapped gift, and presents it to the second player.
- The second player takes it, unwraps its elaborate imaginary packaging, announces what it is, and thanks the first player.
- The first player has to accept whatever the second player says the gift is and explain why it was specially chosen for the recipient.

After playing this game, one teacher told her class that "a conjecture is like a gift" (Knudsen, et al, 2014, 2017). She said that when you get a new conjecture, it may be unfamiliar, but you don't reject it. You take it and see how you can work with it. In this way, playing and discussing the game helps students internalize the classroom norm, "Accept all conjectures in the conjecturing phase of argumentation."

Establishing norms through improv games is also a practice for fostering classroom-level equity. Socio-mathematical norms might be already known and practiced by some members of the class, but not by others (Herbel-Eisenmann., Choppin, Wagner, & Pimm, 2011))—in many cases BIPOC youth, whose experiences in argumentation, while no less relevant to the classroom community, may come with different norms.

* This game can be found in Knudsen (2017). This and many improv games were drawn from the collective corpus that improvisational actors use to learn their craft.



Mathematical Argumentation in Middle School—the What, Why, and How **is available from Corwin at https://qrgo.page.link/fpeCc**

The model of argumentation and PD model summarized here were developed over 15 years of research, teacher interactions, and classroom observation. Our current project, *Visualize Teaching*, will use both these models but extend them to content coaching and to more mathematical practices, such as seeing and using structure. *Visualize Teaching* also includes video-based coaching, where teachers capture video of their class engaged in argumentation and discuss it with their coaches, providing a clear window onto their practices. The first *Visualize Teaching PD* is coming up in the summer of 2021, and we are excited to see how teachers and coaches work together with these tools. Learn more at terc.edu/viste/

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Jennifer Knudsen is a mathematics educator whose work is at the crossroads of research and design. A veteran curriculum and professional development designer, Knudsen has led many projects, from mathematical argumentation to 3D modeling and printing for learning mathematics and computational thinking.

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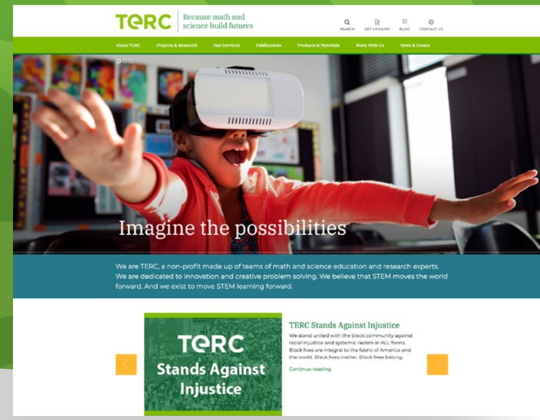

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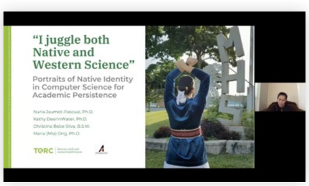

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